

# Introduction to the Mathematical and Statistical Foundations of Econometrics

## *Errata to the first edition*

April 5, 2007

Page 10, line 2 from bottom:	Replace “smallest” with “largest”
Page 10, line 1 from bottom:	Replace $\geq$ with $\leq$
Page 20, line 5 from bottom:	Replace $P(\{T\})$ with $P(\{T\})$
Page 37, line 2 of Section 2.1:	Replace “dice” <sup>1</sup> with “a die”
Page 37, line 4 of Section 2.1:	Replace “dice” with “die”
Page 38, line 16 from top:	Replace “dice” with “die”
Page 49, line 10 from top:	Replace $\mu X(B_j)$ with $\mu_X(B_j)$
Page 50, line 1 from top:	Replace “Chapter 7” with “Chapter 6”
Page 53, line 10 from bottom:	Replace $X = U_0 \cdot U_2$ with $Y = U_0 \cdot U_2$
Page 84, last equation:	Replace the second <b>and</b> last integrals with $\int_{A \cap B_m} \lim_{n \rightarrow \infty} Z_n^{(m)}(\omega) dP(\omega)$
Page 89, line 2 from bottom:	Replace “measure” with “measurable”
Page 90, lines 6 and 7 from top:	Replace the function $g(x)$ with $g(x) = \sin^2(\pi x/2) = (\sin(\pi x/2))^2$
Page 113, line 2 in Theorem 5.2:	Replace “characteristic of $Y$ ” with “characteristic function of $Y$ ”
Page 131, line 3 of Section 5.7.3:	Replace “have” with “has”
Page 144, line 1 of Theorem 6.7:	Replace “Let $X_n$ a sequence” with “Let $X_n$ be a sequence”
Page 146, first line after eq. (6.8):	Replace “Theorem 6.3” with “Theorem 6.10”
Page 150, line 7 from top:	Replace $X_n \rightarrow_p \cdot$ with $X_n \rightarrow_p X$ .
Page 179, line 1 from top:	Replace “Chapter 6 I focused” with “Chapter 6 focused”
Page 185, line 11 from top:	Replace “shock” with “shocks”
Page 185, line 3 of Theorem 7.5:	Replace $\mathcal{F}_{-\infty} = \bigcap_{t=1}^{\infty} \mathcal{F}_{-\infty}^t$ with $\mathcal{F}_{-\infty} = \bigcap_t \mathcal{F}_{-\infty}^t$
Page 188, equation (7.22):	Replace $U_t$ with $X_t$
Page 188, line 7 from below:	Replace “ $t < n$ ,” with “ $t < 0$ as well,”

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<sup>1</sup> Note that “dice” is the plural of “die”.

Page 190, last equation:

Replace this equation with

$$\partial f_t(\theta_0)/\partial \theta_0^T = \begin{pmatrix} \sum_{j=1}^{\infty} (\beta_0 + (\beta_0 - \gamma_0)(j-1)) \beta_0^{j-2} V_{t-j} \\ -\sum_{j=1}^{\infty} \beta_0^{j-1} V_{t-j} \end{pmatrix}$$

Page 191, equation (7.35):

Replace  $\beta_0^{2(j-2)}$  in the expressions of the upper-right and lower-left elements of the matrix involved with  $\beta_0^{2j-3}$

Page 193, Lemma 7.2:

Replace “of random ” with “of zero-mean random ”

Page 196, line 1 after (7.58):

Replace “ $\sigma$ -mixing” with “ $\alpha$ -mixing”

Page 200, Definition 7.A.2:

Replace  $X_n$  with  $x_n$

Page 201, line 4 from top:

Replace  $n, m \rightarrow \infty$  with  $k, m \rightarrow \infty$

Page 201, line 12 from top:

Replace the sentence “Moreover, it is easy to verify that  $E[X] = 0$  and  $E[X^2] < \infty$ .” with the following:

To prove that  $E[X^2] < \infty$ , observe that for any  $K > 0$  and fixed  $m$ ,

$$\begin{aligned} E[X^2 I(|X| < K)] &= E[(X - X_m)^2 I(|X| < K)] + 2E[(X - X_m)X_m I(|X| < K)] + E[X_m^2 I(|X| < K)] \\ &\leq E[(X - X_m)^2] + 2\sqrt{E[(X - X_m)^2]E[X_m^2]} + E[X_m^2] = \|X - X_m\|^2 + 2\|X - X_m\|\sqrt{E[X_m^2]} + E[X_m^2] < \infty. \end{aligned}$$

Letting  $K \rightarrow \infty$  it follows from this inequality and the monotone convergence theorem that  $E[X^2] < \infty$ . To prove  $E[X] = 0$ , observe that

$$|E[X]| = |E[X] - E[X_m]| \leq E[|X - X_m|] \leq \sqrt{E[(X - X_m)^2]} = \|X - X_m\| \rightarrow 0 \text{ as } m \rightarrow \infty,$$

where the first equality follows from the fact that  $E[X_m] = 0$ , and the second inequality follows from Liapounov’s inequality.

Page 202, last line:

Replace  $1/n^2$  with  $2/n^2$ , and  $1/m^2$  with  $2/m^2$

Page 203, line 10 from bottom:

Replace  $\|Y_t\|^2 = E[Y_t^2] < \infty$  with  $\|\hat{X}_t\|^2 = E[\hat{X}_t^2] < \infty$

Page 204, line 1 from top:

Replace  $Z_{t,m} = \sum_{j=1}^m \alpha_j U_{t-j}$  with  $Z_{t,m} = \sum_{j=0}^m \alpha_j U_{t-j}$

Page 204, line 1 from top:

Replace  $\alpha_j = \langle X_t, U_{t-j} \rangle = E[X_t U_{t-j}]$  with  $\alpha_j = \langle X_t, U_{t-j} \rangle / \sigma_u^2 = E[X_t U_{t-j}] / \sigma_u^2$

Page 204, lines 2-4 from top:

Replace the equation with

$$\begin{aligned}\|X_t - Z_{t,m}\|^2 &= \|X_t - \sum_{j=0}^m \alpha_j U_{t-j}\|^2 \\ &= E[X_t^2] - 2 \sum_{j=0}^m \alpha_j E[X_t U_{t-j}] + \sum_{i=0}^m \sum_{j=0}^m \alpha_i \alpha_j E[U_i U_j] \\ &= E[X_t^2] - \sigma_u^2 \sum_{j=0}^m \alpha_j^2 \geq 0\end{aligned}$$

Page 204, line 5 from top:

Replace  $\sum_{j=1}^{\infty} \alpha_j^2 < \infty$  with  $\sum_{j=0}^{\infty} \alpha_j^2 < \infty$

Page 204, line 6 from top:

Replace  $S_{-\infty}^{t-1}$  with  $S_{-\infty}^t$

Page 204, line 7 from top:

Replace  $Z_t = \sum_{j=1}^{\infty} \alpha_j U_{t-j} \in S_{-\infty}^{t-1}$  with  $Z_t = \sum_{j=0}^{\infty} \alpha_j U_{t-j} \in S_{-\infty}^t$

Page 204, line 8 from top:

Replace  $W_t = X_t - \sum_{j=1}^{\infty} \alpha_j U_{t-j}$  with  $W_t = X_t - \sum_{j=0}^{\infty} \alpha_j U_{t-j}$

Page 207, line 3 from bottom:

Replace  $E[L_1(\theta)/L_1(\theta_0)|\mathcal{F}_0]$  with  $E[\hat{L}_1(\theta)/\hat{L}_1(\theta_0)|\mathcal{F}_0]$

Page 218, equation (8.36):

Replace = with = -

Page 218, equation (8.38):

Replace = - with =

Page 223, last equation:

Replace = - with =

Page 224, equation (8.58):

Replace = - with =

Page 224, equation (8.59):

Replace = - with =

Page 244, lines 2 and 3:

The statement “Moreover, if  $E$  is an elementary matrix and  $P_{ij}$  is a elementary permutation matrix then  $P_{ij}E = EP_{ij}$ .” is false.<sup>2</sup> Therefore, this sentence has to be deleted.

Page 244, Theorem I.7:

Consequently, the first part of Theorem I.7 is false. This theorem has to be replaced with the following:

**Theorem I.7:** Let  $P_{ij}$ ,  $i \neq j$ , be an elementary permutation matrix and let  $E_{k,m}(c)$  be a conformable elementary matrix. Then

(a)  $i \notin \{k,m\}$  and  $j \notin \{k,m\}$  imply  $P_{ij}E_{k,m}(c) = E_{k,m}(c)P_{ij}$ ,

(b)  $i = k$  and  $j \neq m$  imply  $P_{ij}E_{k,m}(c) = E_{j,m}(c)P_{ij}$ ,

(c)  $i \neq k$  and  $j = m$  imply  $P_{ij}E_{k,m}(c) = E_{k,i}(c)P_{ij}$ ,

(d)  $i = k$  and  $j = m$  imply  $P_{ij}E_{k,m}(c) = E_{m,k}(c)P_{ij}$ .

Consequently, for every elementary matrix  $E$  and conformable permutation matrix  $P$  there exists an

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<sup>2</sup> Thanks to Christian Kascha for pointing this problem out to me.

elementary matrix  $E_*$  such that  $PE = E_*P^T$ . Moreover,  $P^{-1} = P^T$ .

*Proof:* Consider the matrix  $P_{i,j}E_{k,m}(c)P_{i,j}$ , which is the results of swapping rows  $i$  and  $j$  of  $E_{k,m}(c)$  first and then swapping columns  $i$  and  $j$ . In case (a) the position of  $c$  remains the same, hence  $E_{k,m}(c) = P_{i,j}E_{k,m}(c)P_{i,j}$ , which implies that  $P_{i,j}E_{k,m}(c) = E_{k,m}(c)P_{i,j}$  because  $P_{i,j}P_{i,j} = I$ . In case (b), swapping rows  $k$  and  $j$  of  $E_{k,m}(c)$  moves  $c$  to position  $(j,m)$ , and subsequent swapping of columns  $k$  and  $j$  leaves  $c$  in the new position  $(j,m)$ , because  $m \neq j$  and  $m \neq k$ , but restores the elementary matrix structure. Therefore, in this case  $E_{j,m}(c) = P_{i,j}E_{k,m}(c)P_{i,j}$ . In case (c), swapping rows  $i$  and  $m$  of  $E_{k,m}(c)$  leaves  $c$  in position  $(k,m)$ , and subsequent swapping of columns  $i$  and  $m$  moves  $c$  to the new position  $(k,i)$  but the restores the elementary matrix structure. In case (d), swapping rows  $k$  and  $m$  of  $E_{k,m}(c)$  moves  $c$  to position  $(m,m)$ , and subsequent swapping of columns  $k$  and  $m$  moves  $c$  to position  $(m,k)$ . Q.E.D.

Page 245, lines 8 & 7 from bottom: Replace the two lines between equations (I.25) and (I.26) with:  
 “for instance. It follows now from Theorem I.7(b), equation (I.25) and the trivial equality  $P_{ij} = P_{ji}$  that”

Page 245, equation (I.26): Replace this equation with:

$$\begin{aligned}
 P_{2,3}E_{3,1}(\frac{1}{2})E_{2,1}(-\frac{1}{2})A &= P_{3,2}E_{3,1}(\frac{1}{2})E_{2,1}(-\frac{1}{2})A \\
 &= E_{2,1}(\frac{1}{2})P_{2,3}E_{2,1}(-\frac{1}{2})A \\
 &= E_{2,1}(\frac{1}{2})E_{3,1}(-\frac{1}{2})P_{2,3}A \\
 &= DU.
 \end{aligned} \tag{I.26}$$

Page 245, last equation: Replace this equation with

$$E_{2,1}(\frac{1}{2})E_{3,1}(-\frac{1}{2}) = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix}$$

Page 246, equation (I.27):

Replace this equation with:

$$\begin{aligned} (E_{2,1}^{(1/2)}E_{3,1}^{(-1/2)})^{-1} &= \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ -0.5 & 0 & 1 \end{pmatrix}^{-1} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ -0.5 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix} = L \end{aligned} \tag{I.27}$$

Page 287, line 2 from bottom:

Replace the inequality with

$$\varphi(\lambda a + (1 - \lambda)b) \leq \lambda\varphi(a) + (1 - \lambda)\varphi(b)$$

Page 288, line 2 from top:

Replace the inequality with

$$\varphi(\lambda a + (1 - \lambda)b) \geq \lambda\varphi(a) + (1 - \lambda)\varphi(b)$$

Page 295, first **and** last equations:

Replace  $x_n$  with  $x_0$