Money as a Mechanism in a Bewley Economy∗

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Abstract

We study what features an economic environment might possess, such that it would be Pareto efficient for the exchange of goods in that environment to be conducted on spot markets where those goods trade for money. We prove a conjecture that is essentially due to Bewley [1980, 1983]. The gist is that monetary spot trading is nearly efficient ex ante in an environment where very patient agents can accumulate large enough money stocks to be completely self insured. We also study examples where a nonmonetary mechanism is preferred ex ante to any monetary mechanism (at least, when the comparison is restricted to stationary equilibria) and where an inflationary monetary mechanism is preferred ex ante to a laissez-faire or deflationary monetary mechanism in an environment with impatient agents.

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1. Introduction

The 1980 “Models of Monetary Economies” volume was a milestone in a change of thinking about the nature of money and its role in the economy. The former way of thinking had emphasized that money is a quantity or set of quantities (of currency, demand deposits, bank reserves, and so forth). Since 1980, it has become common to think of money as an institution or system of institutions, and to model the necessity of such institutions. A formal theory of economic institutions (or “mechanisms”), which came to maturity at about this same time, was progressively recognized as a framework for such modelling. Mechanism design is figuratively described in terms of an incompletely informed planner’s problem of allocating resources on the basis of agents’ communication (not necessarily truthful) about their private, welfare-relevant information. Researchers such as Kocherlakota and Wallace [1998] have used the mechanism-design framework explicitly in the past several years to address welfare economic questions about monetary institutions in a clear, consistent way.

Ironically, many researchers have become skeptical about the classical mechanism-design framework because it has been shown to be possible to implement, virtually exactly, practically any allocation that is technologically feasible. Such virtual-implementation results, in combination with the strong intuition that some desirable, technologically feasible allocations are actually impossible to implement, suggest that there are important constraints on implementation in actual economies that the classical framework ignores.

Our perspective is that the logic of studying monetary institutions in terms of implementation is valid, but that this logic must be applied to a formal economic environment with appropriately modelled constraints on mechanism design. We carry out this program for a version of the model economy that Truman Bewley introduced in the 1980 volume to analyze the welfare economics of taxation or subsidy of money holdings. We emphasize three constraints. First, each agent’s endowment and preferences among net trades are private information (as in Bewley [1980, 1983], Atkeson and Lucas [1992], and Levine and Zame [2002]). Second, agents can consume their own endowments without restriction or nonpecuniary punishment (as in Kocherlakota [2002]). Third, each agent is partially anonymous in the sense that the mechanism can make use only of summary information about his history and current state (also imposed in Kocherlakota [2002]). We investigate the circumstances under which a mechanism that resembles monetary transactions may, or may not, be efficient or nearly efficient. By efficiency, we mean that a mechanism weakly
implements an allocation that is Pareto efficient within the set of all mechanisms that satisfy the three constraints.\footnote{Weak implementation means that the allocation in question is among those resulting from equilibria (specifically, in this study, among the stationary Bayesian Nash equilibria) of the mechanism. The mechanism need not have a unique equilibrium, though.}

Bewley formulates a finite-agent economy with idiosyncratic, transient shocks and competitive spot trading.\footnote{There are other relevant considerations, such as there being only a single commodity in each goods-for-money spot market.} He conjectures that, by using a nominally denominated, fiat asset, sufficiently patient agents can insure themselves almost completely without forward trading or contingent claims trading. Generally, in order for them to do so, money must have a rate of return that is close to the agents' rate of time preference. When agents are patient, this means that the interest rate on money must be approximately zero.\footnote{In [1980], Bewley conjectures that full risk sharing is achieved in the limit if the gross interest rate is set equal to the inverse of agents' discount factor. In [1983], he shows that setting the interest rate exactly at that level generally precludes equilibrium from existing, but that the interest rate on money can be set arbitrarily close to the inverse of the discount factor as the discount factor approaches unity.} (Bewley points out that this is a special case of “Friedman's rule,” which advocates mild deflation.) Bewley's results, and his discussion of them, also make it clear that trade in a noninflationary monetary regime does not achieve full insurance if traders are impatient (or shocks are permanent) and risk averse. We formulate and prove Bewley's conjecture, and also provide an example of how an inflationary monetary mechanism can be efficient in a Bewley environment populated by impatient, risk-averse agents.\footnote{There is a small body of literature on the potential beneficial effect of social insurance that inflationary monetary policy can provide in these circumstances. Levine [1991] and Kehoe, Levine and Woodford [1992] study two-state Markov equilibrium in an environment where two types of agents switch their preferences stochastically and equilibrium distribution of money balances is degenerate. Deviatov and Wallace [2001] study the issue in a search theoretic model of money where money is indivisible and agents can hold at most two units of it. Molico [1997] and Edmond [2002] provide numerical examples of inflationary monetary policy dominating other policies, the former in a random-matching model of money, and the latter in an over-lapping generation setting with money-in-utility-function.}

As a competitive equilibrium model, Bewley's model implicitly satisfies the three constraints that we have just mentioned. A limitation of the competitive-equilibrium formulation is that the standard welfare theorems compare equilibrium allocations to hypothetical “first-best” allocations that would be technologically feasible but that might not be implementable by any mechanism that satisfies the three constraints that we impose. The advantage of the mechanism design framework that we adopt is that the equilibrium allocation of the regime in question is compared directly and explicitly to equilibrium allocations of other monetary regimes and nonmonetary mechanisms, rather than to a hypothetical
allocation that might not be an equilibrium allocation of any feasible mechanism.

We refer the reader to Bewley [1980, 1983] for his insightful and balanced discussion of both the significance and the limitations of his model within monetary theory. This is a partial equilibrium model in several respects. One is that the fiat asset called ‘money’ is the only durable asset, so the model would have to be extended to explain why agents would willingly hold this asset if another asset had a higher rate of return. Another is that monetary trade is posited to be absolutely the only means of state-contingent or intertemporal trade. Obviously there are some markets for such trades in an actual economy, particularly (although not exclusively) for trades contingent on various types of aggregate shock or fluctuation. Allocations in Bewley’s original model of finitely many traders with idiosyncratic shocks necessarily involve aggregate shocks because the sum of finitely many independent random variables is random. However, the point of the model is that money holdings can serve to insure individuals against idiosyncratic shocks for which no markets exist, rather than that a monetary mechanism might make markets redundant for insuring aggregate shocks. Here we considerably simplify the proof of the main result in Bewley [1983] by considering a continuum-of-agents model without aggregate risk. We call this a Bewley model because our amendment does not compromise its suitability to address the issues that the finite-agent model was formulated to study.

2. The environment

The economy is an infinite horizon exchange economy. Time is discrete and denoted by $t = 0, 1, 2, \ldots$. There is a continuum $(I, \mathcal{I}, \mu)$ with measure 1 of infinite-lived agents. At each date, there is a single perishable good with which agents are endowed, and that they trade and consume.

Agents’ endowments and preferences fluctuate. For a generic agent $i$, his date- $t$ state $\theta_{it}$ is a sequence of independent, identically distributed random variables taking values in a finite state space $\Theta$. Each $\theta_{it}$ has distribution $\pi$ on $\Theta$. Each agent’s state follows his own independent process. We assume that the realization of the sequence of profiles of individual

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5 $\mathcal{I}$ is a $\sigma$-algebra of subsets of $I$, and $\mu$ is a measure defined on those subsets, such that $\mu(I) = 1$.

6 Bewley [1980, 1983] and Levine and Zame [2002] model each agent’s shocks as Markovian, and study price-taking equilibrium in an economy with finitely many traders. Bewley assumes time is infinite in the past as well as the future, which avoids there being an initial condition and ensures existence of a stationary equilibrium. Levine and Zame study an equilibrium that is not stationary in general. In the markovian case, the stationary joint distribution of money balances and individual shocks is statistically dependent. Since we will treat the initial distribution of money balances as part of the mechanism, and since we confine
agents’ states \( \{ \theta_{it} \}_{i \in I, t=0}^{\infty} \) is an i.i.d. process of random variables with distribution \( \pi \).

Letting \((\Omega, \mathcal{B}, P)\) denote the probability space on which all of these random variables are defined, we assume an idealized “law of large numbers” that for every \( i \in I, \omega \in \Omega, \theta \in \Theta \) and \( t \in \mathbb{N} \),

\[
\mu(\{ j \mid \theta_{jt}(\omega) = \theta \}) = P(\{ \zeta \mid \theta_{it}(\zeta) = \theta \}).
\]

At each date, an agent with state \( \theta \in \Theta \) receives endowment \( e(\theta) \) and enjoys period utility \( u(c, \theta) \) if he consumes \( c \) units of good. The endowment good is perishable. \( \mathbb{E}[e(\theta)] > 0 \). The consumption set at each date, and on each sample path, is the set \([0, \infty)\) of nonnegative real numbers. The bounded function \( u: \mathbb{R}_+ \times \Theta \rightarrow [0, b] \) is weakly increasing, continuous, and concave in \( c \). It is assumed that, when \( u(c, \theta) \) is regarded as a function of \( c \), it is concave and strictly increasing. Agents maximize the discounted expected utility of their future consumption streams, with common discount factor \( \beta \).

Agents exchange endowments according to a trading mechanism that must be feasible with respect to some informational constraints in the environment. Competitive trading using money can be implemented by a mechanism that meets these constraints. First we discuss the constraints and define a trading mechanism in general terms, and then we will specify the mechanism that implements competitive monetary trade.

Each agent \( i \) privately learns his own realization of \( \theta_{it} \) at date \( t \). Each agent \( i \) delivers a quantity \( z_{it} \in \mathbb{R}_+ \) of the endowment good to a resource pool at the planner’s disposition and also sends a message \( m_{it} \in \mathbb{R} \) to the planner.\(^8\) The planner maintains a one-dimensional summary statistic (that is, a real number) \( w_{it} \) regarding \( i \)’s history, as will be described fully below. The planner uses the summary statistics and messages of all agents and the amounts contributed by all agents to update the summary statistic of each agent \( i \) and to reallocate a quantity \( y_{it} \) of the endowment good from the resource pool to \( i \). Agent \( i \) consumes \( c_{it} = e(\theta_{it}) - z_{it} + y_{it} \). The quantities \( w_{it}, m_{it}, z_{it}, y_{it}, c_{it}, \) and \( w_{i(t+1)} \) are observed by agent \( i \) and the planner, but not by the other agents.

The planner’s limited memory and the agents’ inability to observe or communicate with one another are important features of the environment. The planner is not able to recall the entire history of his dealings with agent \( i \) prior to date \( t \), but only the one-dimensional statistic \( w_{it} \). Because the agents are ignorant of other’s histories, states and reports, which

\(^7\)Green [1994] examines this probability model in detail.

\(^8\)As is conventional in the study of an economy with a continuum of agents, the profiles of these quantities and messages will formally be represented by measurable functions. This formalization is made explicit below.
are reflected in the planner’s decisions, in principle an agent might draw inferences about other agents from observing the planner’s decisions. Although the stationarity and “law-of-large-numbers” assumptions regarding the particular environment studied here make such inference uninformative (when the mechanism treats all agents symmetrically and in a time-invariant way), for logical clarity we will not suppress past decisions of the planner as arguments of an agent’s decision rule.

Another feature that we emphasize heavily (following Kocherlakota [2002]) is the planner’s limited enforcement power. The planner cannot impose any nonpecuniary penalty on an agent for sending or failing to send a particular message, or for not following an instruction given in the planner’s message. The worst that the planner can do is to give the agent nothing in the current period when the endowment pool is reallocated, and then update the agent’s summary statistic to a value that encodes the fact that the prohibited message has been sent or that the instruction has been flouted, and then to treat the agent ungenerously in the future as a result of the summary statistic having that unfavorable value. In particular, the worst outcome that the planner can impose on an agent is autarky. (The planner would impose autarky on agent $i$ by setting $y_{it} = 0$ for the current and all future $t$. Faced with this planner’s policy, $i$ would optimally set $z_{it} = 0$ for all future $t$.)

We will denote the set of profiles of summary statistics of all agents by $F$, the set of profiles of agents’ contributions to the resource pool by $P$, and the set of profiles of agents’ messages to the planner by $G$. Formally, let $F$ be the set of measurable functions from $I$ to $\mathbb{R}$, let $P$ be the set of nonnegative-valued functions in $F$, and let $G = F$. If $f \in F$, then we use $f_i$ to denote $f(i)$, and so forth with elements of other spaces of functions on $I$. A trading mechanism consists of an initial $w_0 \in F$ and time-indexed sequences of updating rules $W = \{W_t : F \times G \times P \rightarrow F\}_{t \in \mathbb{N}}$ and reallocation rules $Y = \{Y_t : F \times G \times P \rightarrow P\}_{t \in \mathbb{N}}$. We assume that the planner is able to assign $w_0$ according to any distribution in a way that is independent of all $\theta$, considered as a random variables defined on $(I, \mathcal{I}, \mu)$, almost surely with respect to $(\Omega, \mathcal{B}, \mathbb{P})$.

$^9$An agent can report a real number to the planner. Alternatively, if $\mathbb{R}$ is mapped onto $\Theta$, then the mapping provides a semantics by which an agent can report his current state. Green [1994] provides the relevant measure theory.

$^{10}$Formally, we require that the planner observes a uniformly distributed r.v. $U : I \rightarrow [0, 1]$ such that, for every $\omega$ in an event $B \in \mathcal{B}$ with $P(B) = 1$, the following condition holds. For every probability measure $\psi$ on $\mathbb{R}$, interval $[a, b] \subseteq [0, 1]$, $n \in \mathbb{N}$, and every mapping $f : \{0, \ldots, n\} \rightarrow \Theta$, $\mu(\{i | U(i) \leq b\} \cap \bigcap_{m \leq n}(\{i | \theta_{f(m)}(\omega) = g(m)\})) = (b - a)\Pi_{m \leq n}\pi(\{g(m)\})$. Then, given an arbitrary measure $\psi$ on $\mathbb{R}$ that the planner wants to make the distribution of $w_0$ and letting $f$ be the c.d.f. of $\psi$, he can define $w_0 = \min\{x | U(i) \leq f(x)\}$. 

5
of agents’ summary statistics, messages, and endowment contributions at date $t$; and if $w_{t+1} \in F$ and $y_t \in P$ are the profiles of the planner’s updated summary statistics for the agents and reallocations of endowment to them; then $w_{t+1} = W_t(w_t, m_t, z_t)$, and $y_t = Y_t(w_t, m_t, z_t)$. The reallocation rule $Y_t$ must satisfy the materials-balance condition that $\int_I Y_{jt}(w_t, m_t, z_t) \, d\mu \leq \int_I z_{jt} \, d\mu$.

Agent $i$’s strategy consists of time-indexed sequences of functions $M = \langle M_{it} \rangle_{t \in \mathbb{N}}$ and $Z = \langle Z_{it} \rangle_{t \in \mathbb{N}}$ that specify $i$’s message and the quantity of the endowment good that he delivers, respectively, at date $t$. Agent $i$ has full recall of his own history, including the histories of his states, the values of the summary statistic that the planner has assigned him, and his endowment-good deliveries and messages to the planner. Because $i$ can recursively reconstruct his past deliveries and messages from the other data, those past actions do not have to be explicit arguments of his current decision functions. We can thus represent $M_{it}: (\mathbb{R} \times \Theta)^{t+1} \to \mathbb{R}$ and $Z_{it}: (\mathbb{R} \times \Theta)^{t+1} \to \mathbb{R}_+$. That is, $m_{it} = M_{it}(w_{i0}, \theta_{i0}, \ldots, w_{it}, \theta_{it})$ and $z_{it} = Z_{it}(w_{i0}, \theta_{i0}, \ldots, w_{it}, \theta_{it})$. There is a feasibility constraint that $i$ cannot deliver more than his endowment, that is, $Z_{it}(w_{i0}, \theta_{i0}, \ldots, w_{it}, \theta_{it}) \leq e(\theta_{it})$.

Now we represent a competitive trading arrangement using a constant nominal stock of fiat money as such a mechanism. We suppose that agents hold money as account balances rather than as physical inventories of a fiat object. Indeed, an agent’s money wealth (that is, the amount of money in his account) is the summary statistic that the planner will initially assign and subsequently update. We require that $\forall i \in I w_{i0} \geq 0$ and that $\int_I |w_{i0}| \, d\mu < \infty$.

At every date $t$, the planner essentially operates a spot market according to the rules of a Shapley-Shubik [1977] trading game. The planner interprets each agent’s message as a bid to spend money to acquire other traders’ endowment, disregarding messages that are negative or that exceed the sender’s balance. That is, the planner considers $b_{it} = \max(0, \min(m_{it}, w_{it}))$ to be the money bid of agent $i$. When both the total money bids $\int_I b_{jt} \, d\mu$ and the total goods contribution $\int_I z_{jt} \, d\mu$ are strictly positive, they determine the spot price $p_t = \frac{\int_I b_{jt} \, d\mu / \int_I z_{jt} \, d\mu}$. In such a case, the planner redistributes $b_{it}/p_t$ quantity of the endowment pool to each trader $i$ and adds $p_t \cdot z_{it} - b_{it}$ to the wealth $w_{it}$ of agent $i$. If either $\int_I b_{jt} \, d\mu$ or $\int_I z_{jt} \, d\mu$ is zero, the situation is interpreted as no trade. Hence no goods are distributed, and each agent’s money wealth stay the same. Formally, if we represent the profile of $b_{it}$ by defining $B: F \times G \to P$ according to $\forall i \ B_i(w, m) = \max(0, \min(m_i, w_i))$, ...
then,

\[
Y_{it}(w_t, m_t, z_t) = \begin{cases} 
B_i(w_t, m_t) \frac{\int_I z_{jt} \, d\mu}{\int_I B_j(w_t, m_t) \, d\mu} & \text{if } \int_I z_{jt} \, d\mu \neq 0 \text{ and } \int_I B_j(w_t, m_t) \, d\mu \neq 0 \\
0 & \text{otherwise}
\end{cases}
\]

\[
W_{it}(w_t, m_t, z_t) = \begin{cases} 
\tau Q + (1 - \tau)w_{it} & \text{if } w_{it} \geq 0 \text{ and if } \int_I z_{jt} \, d\mu \neq 0 \text{ and } \int_I B_j(w_t, m_t) \, d\mu \neq 0 \\
\tau Q + (1 - \tau)w_{it} & \text{otherwise}
\end{cases}
\]

It is easy to show that, given \(w_{i0} \geq 0, w_{it} \geq 0\) for all \(t > 0\).

We call a mechanism of this form a \textit{laissez-faire monetary mechanism}, since the planner does not pay interest on money nor tax money nor adjust the nominal money stock after date 0, but merely operates a market on which the agents trade competitively. Note that the specifications of \(Y\) and \(W\) just given are part of the definition of the class of laissez-faire monetary mechanisms. That is, laissez-faire monetary mechanisms differ from one another only in how the initial summary statistics (that is, agents’ initial money balances) \(w_{i0}\) are assigned.

We call a monetary mechanism \textit{stationary inflationary} (resp. \textit{stationary deflationary}) if there is a \(\tau > 0\) (resp. \(\tau < 0\),

\[
\tau Q + (1 - \tau)w_{it} & \text{if } w_{it} \geq 0 \text{ and if } \int_I z_{jt} \, d\mu \neq 0 \text{ and } \int_I B_j(w_t, m_t) \, d\mu \neq 0 \\
\tau Q + (1 - \tau)w_{it} & \text{otherwise}
\]

where \(Q = \int_I w_{jt} \, d\mu\) is the aggregate money balance in the economy. That is, with a stationary inflationary mechanism, an agent’s summary statistics is updated as if his after-trade money holdings is inflated at a constant rate \(\tau\), and the seignorage is distributed as a lump-sum transfer. In contrast, with a deflationary monetary mechanism, an agent’s summary statistics is updated as if he receives interest payment on his money holdings at a rate \(\tau\) which is financed by a lump-sum tax on the population. This lump-sum tax is enforced by the threat of permanent autarky. For an agent \(i\) who has been condemned to autarky by date \(\bar{t}\), \(w_{it} < 0\) for all \(t \geq \bar{t}\).

3. Definition of equilibrium

We focus on symmetric equilibrium, in which all agents use the same strategy \((M, Z)\). (That is, \(M\) and \(Z\) are infinite sequences of functions with the domains and ranges specified
above. Agents may take different actions from one another because their individual states are distinct points of the domains of these decision functions.) A competitive equilibrium is represented by a strategy that each trader is assumed to follow. A strategy is an equilibrium strategy if each agent acts optimally by following it, when he takes it as parametric that the other traders will follow the strategy.

It is well known that such an equilibrium can be characterized by dynamic programming. Consider a mechanism \((w_0, W, Y)\), where each of \(W\) and \(Y\) is a time-indexed sequence of functions. Consider a strategy \((M, Z)\), where each of \(M\) and \(Z\) is a time-indexed sequence of functions, and consider the value function of a trader \(i\) participating in the mechanism, who takes it as parametric that the other traders will all follow \((M, Z)\). Let \(w_t\) be the profile of all agents’ summary statistics at the beginning of date \(t\). For \(j \neq i\), define \(m_{jt} = M_{jt}(w_{j0}, \theta_{j0}, \ldots, w_{jt}, \theta_{jt})\) and \(z_{jt} = Z_{jt}(w_{j0}, \theta_{j0}, \ldots, w_{jt}, \theta_{jt})\). Then define \(m^*(m)\) to be the message profile that results from \(i\) sending message \(m\) while every other agent \(j\) sends the message \(m_{jt}\) specified by strategy \(M\). Formally, define \(m^*: \mathbb{R} \to G\) by \([m^*(m)](i) = m\) and \(\forall j \neq i \quad [m^*(m)](j) = m_{jt}\) and define \(z^*: \mathbb{R} \to P\) by \([z^*(z)](i) = z\) and \(\forall j \neq i \quad [z^*(z)](j) = z_{jt}\). Now the value function \(V^*_t: \mathbb{R} \times F \to \mathbb{R}\) of \(i\) at \(t\) can be defined as

\[
V^*_t(w_{it}, w_t) = \mathbf{E} \left[ \max_{z, m} \left\{ u(e(\theta) - z + [Y_{it}(w_{it}, m^*(m), z^*(z))], \theta) + \beta V^*_{t+1}(w_{it}, m^*(m), z^*(z)) \right\} \right].
\]

The expectation on the right side is taken with respect to the measure \(\pi\) on \(\Theta\). Standard reasoning about the fixed point of a contraction mapping establishes that the sequence \(V^*_0, V^*_1, \ldots\) is uniquely defined. The initial profile of summary statistics \(w_0\), the sequence of statistic-updating rules \(W_t\), and a strategy \((M, Z)\) determine a sequence of summary statistics \(w_t\). The strategy \((M, Z)\) is an equilibrium strategy if, for all \(t\) and for all \(w\) in the range of \(w_t\), \(Z\) and \(M\) specify the optimizing values of \(z\) and \(m\) in the expression on the right side of the value function.

For an equilibrium strategy \((M, Z)\), define the value function sequence of the equilibrium by \(V_t(w_{it}) = V^*_t(w_{it}, w_t)\). In particular, in the case of a monetary mechanism with stationary policy \(\tau\), \(Y_t\) and \(W_t\) are defined in terms of the price

\[
p_t = \frac{\int_I M_{jt}(w_{j0}, \theta_{j0}, \ldots, w_{jt}, \theta_{jt}) \, d\mu}{\int_I Z_{jt}(w_{j0}, \theta_{j0}, \ldots, w_{jt}, \theta_{jt}) \, d\mu}.
\]

Note that \(p_t\) can be 0, \(\infty\), or undefined (0/0), corresponding to the case of no trade at time \(t\). Utilizing these observations, the value to an agent of having the summary statistic \(w\) at
date $t$ is a function $V_t: \mathbb{R} \to \mathbb{R}$ is defined by

$$V_t(w) = \begin{cases} \mathbb{E}\left[\max_{z \in [0,e(\theta)]} \left\{ u(e(\theta) - z + \frac{m}{p_t}, \theta) + \beta V_{t+1}\left(\tau Q + (1 - \tau)(w + p_t z - m)\right) \right\} \right] & \text{if } 0 < p_t < \infty \\ \mathbb{E}\left[u(e(\theta), \theta) + \beta V_{t+1}\left(\tau Q + (1 - \tau)w\right)\right] & \text{otherwise} \end{cases}$$

(5)

Restricting $m$ to the interval $[0, w_t]$ is justified because the mechanism truncates bids to this interval via $B$.

We conclude this section by defining stationary Markov competitive equilibrium of a monetary mechanism, the existence of which will be investigated in Section 4. Define the current-date projection mapping $\gamma: \bigcup_{t \in \mathbb{N}}(\mathbb{R} \times \Theta)^{t+1} \to \mathbb{R} \times \Theta$ by $\gamma(w_0, \theta_0, \ldots, w_t, \theta_t) = (w_t, \theta_t)$. A sequence $\langle H_t: (\mathbb{R} \times \Theta)^{t+1} \to \mathbb{R} \rangle_{t \in \mathbb{N}}$ is stationary Markov if for each $t$, $H_t = H_0 \circ \gamma$.

An equilibrium $(M, Z)$ is a stationary Markov competitive equilibrium if the sequences $M_i$ and $Z_i$ are stationary Markov for all $i$ and, almost surely with respect to $(\Omega, \mathcal{B}, P)$, $(w_0, \theta_0)$ and $(w_1, \theta_1)$ are identically distributed $\mathbb{R}^2$-valued random variables on $I$. These are sufficient conditions for $\langle w_{it}, \theta_{it}, c_{it}\rangle_{t \in \mathbb{N}}$ to be almost surely a stationary Markov process on $I$ and for the spot price $p_t = \int_I b_{jt} \, d\mu / \int_I z_{jt} \, d\mu$ to be constant over time.\(^{11}\) While a strictly positive, finite price indicates a stationary equilibrium with active trading, a price of zero or infinity or undefined price (0/0) corresponds to autarky, a no-trade equilibrium.

Given any stationary equilibrium with active trade, clearly there is another monetary mechanism for which the time-invariant price is 1 and the equilibrium allocation is identical to that of the original mechanism. The new mechanism is obtained simply by dividing $w_{i0}$ by the equilibrium price $p_0$, for each trader $i$. The equilibrium strategy in the mechanism is obtained from that of the old one by the same normalization. In a stationary Markov competitive equilibrium with price 1, the definition of equilibrium can be simplified by defining the net trade $x_t = z_t - m_t$. Then the Bellman equation can be rewritten as

$$V(w) = \mathbb{E}\left[\max_{x \in [-w, e(\theta)]} \left\{ u(e(\theta) - x, \theta) + \beta V\left(\max(\tau Q + (1 - \tau)(w + x), 0)\right)\right\} \right].$$

(6)

\(^{11}\)Note that the function sequences $W$ and $Y$ of a laissez-faire, stationary inflationary, or stationary deflationary monetary mechanism are stationary Markov. The definition of stationary equilibrium given here is the appropriate definition, in view of this fact. An example of a monetary mechanism that is not itself stationary Markov is one in which each agent receive a so-called “helicopter drop,” that is, a fixed amount of newly created fiat money, proportional to the current aggregate nominal money stock, in each period. The mechanism is not stationary Markov because the amount received, which grows geometrically, is a time-dependent, additively separable term of $W$. The appropriate definition of stationary Markov equilibrium for this mechanism would focus on time invariance of the distribution of agents’ real balances, rather than of their nominal balances.
4. Existence of a laissez-faire monetary mechanism having a stationary Markov competitive equilibrium

In this section we prove that, for any environment satisfying the assumptions in Section 2, there is a laissez-faire monetary mechanism that has a stationary Markov competitive equilibrium. This is done by studying an auxiliary optimization problem of an autarkic agent who can store the endowment good without depreciation, and by applying information about the solution of this problem to construct the equilibrium.

Consider an environment identical to that of Section 2 except in three respects: there is only one agent rather than a continuum, he receives an endowment of size $w_0 + e(\theta_0)$ at date 0, and he can store without depreciation the endowment that he has received. Other aspects of the model are the same. That is, the agent’s endowment and utility are functions of an i.i.d. process $\langle \theta_t \rangle_{t \in \mathbb{N}}$ taking values in a finite set $\Theta$ and having distribution $\pi$. He receives endowment $w_0 + e(\theta_0)$ at date 0 and $e(\theta_t)$ at each date $t > 0$. The agent chooses date-0 consumption $c_0$ from $[0, w_0 + e(\theta_0)]$ and, for $t > 0$, chooses date-$t$ consumption $c_t$ from $[0, w_t + e(\theta_t)]$ (where $w_t = w_{t-1} + e(\theta_{t-1}) - c_{t-1}$) as a function of previous history. He maximizes expected discounted utility $E\left[ \sum_{t \in \mathbb{N}} \beta^t u(c_t, \theta_t) \right]$, and his utility function $u(c, \theta)$ is bounded, and strictly increasing and concave in $c$.

Standard dynamic programming results (cf. Lucas and Stokey [1989]) provide the following information.

**Lemma 1.** For the auxiliary problem, there is a decision function $C: \mathbb{R}_+ \times \Theta \to \mathbb{R}_+$ such that the agent’s optimal choice at every date $t$ is that $c_t = C(w_t, \theta_t)$. There is a strictly concave, increasing value function $V: \mathbb{R}_+ \to [0, b/(1 - \beta)]$ such that, for all $w$ and $\theta$,

$$C(w, \theta) = \arg \max_{c \in [0, w + e(\theta)]} \left[ u(c, \theta) + \beta V(w + e(\theta) - c) \right]$$

and $V(w) = E[u(C(w, \theta), \theta) + \beta V(w + e(\theta) - C(w, \theta))]$. There is a probability measure $\psi$ on $\mathbb{R}_+$ such that $\langle (w_t, \theta_t) \rangle_{t \in \mathbb{N}}$ is a Markov process that has stationary transition probabilities and that converges weakly to a stationary asymptotic distribution such that the marginal distribution of $w$ is $\psi$.

For this specific optimization problem, Lemma 1 can be sharpened by showing that $\psi$ has bounded support.

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12The proof can be easily extended to the case of stationary inflationary monetary mechanism.
Lemma 2. For the stationary asymptotic marginal distribution \( \psi \) of Lemma 1, there exists \( \bar{w} \in \mathbb{R}_+ \) such that \( \psi([0, \bar{w}]) = 1 \).

Proof. Since \( V \) is concave, for every \( w \in \mathbb{R}_+ \), there is a subgradient, that is, a number \( g_w \in \mathbb{R}_+ \) satisfying, for all \( x \in \mathbb{R}_+ \), \( V(x) \leq V(w) + (x-w)g_w \). Setting \( x = 0 \) and noting that \( 0 \leq V(0) \leq V(w) \leq b/(1-\beta) \), the subgradient inequality yields \( g_w \leq b/(w(1-\beta)) \). For each \( \theta \in \Theta \), consider \( u(c, \theta) \) as a function of \( c \) and let \( h_\theta \in \mathbb{R}_+ \) be a subgradient of the function at \( e(\theta) \). If \( \bar{w} > \beta b/(1-\beta) \min_{\theta \in \Theta} h_\theta \), then equation (7) implies that \( C(w, \theta) > e(\theta) \) for all \( w \geq \bar{w} \) and for all \( \theta \). Thus \( w_t > \bar{w} \) implies that \( w_{t+1} < w_t \). Equation (7) also shows, in conjunction with the fact (established in Rockafellar [1970], Theorem 24.3) that every selection from the subdifferential of a continuous concave function is nonincreasing, that \( w_t \leq \bar{w} \) implies \( w_{t+1} \leq \bar{w} \). That is, \( w_t \) first decreases monotonically to a level not exceeding \( \bar{w} \) if \( w_0 > \bar{w} \), and then does not escape from the interval \([0, \bar{w}]\). Therefore, since \( \psi \) is the marginal of a stationary distribution, \( \psi([0, \bar{w}]) = 1 \). □

Now we apply this information regarding solution of the auxiliary problem to specifying a laissez-faire monetary mechanism that has a stationary Markov competitive equilibrium.

Proposition 1. In an environment such as has been described in Section 2, and where the utility function \( u \) is strictly concave in \( c \) for each \( \theta \), there is a laissez-faire monetary mechanism that has a stationary Markov competitive equilibrium.

Proof. This mechanism is specified by distributing \( w_0 \) according to the stationary marginal distribution \( \psi \) in the solution of the auxiliary problem. Clearly \( \psi \) has finite mean, since \( \mu \) is a finite measure and \( \psi \) has bounded support by Lemma 2. The agents’ stationary strategy is defined in terms of the decision function \( C \) of Lemma 1. Specifically for every agent \( i \), \( M_{it}(w_{i0}, \theta_{i0}, \ldots w_{it}, \theta_{it}) = \max(0, C(w_{it}, \theta_{it}) - e(\theta_{it}) \) and \( Z_{it}(w_{i0}, \theta_{i0}, \ldots w_{it}, \theta_{it}) = \max(0, e(\theta_{it}) - C(w_{it}, \theta_{it})) \). By induction on \( t \), the joint distribution of \( w_t \) and \( \theta_t \) (as random variables on \((I, \mathcal{I}, \mu)\)) is the same as the stationary distribution of \( w \) and \( \theta \) in the auxiliary problem.\(^{13}\) Thus, by stationarity of that distribution, the equilibrium price \( p_t \) is 1 and the distribution of \( w_{t+1} \) is also \( \psi \). Since \( p_t = 1 \) for all \( t \) almost surely, the decision problem of an agent in this equilibrium is isomorphic to the agents’ decision problem in the auxiliary problem. Thus \( M \) and \( Z \) are an equilibrium strategy because \( C \) is the optimal strategy in the auxiliary problem. □

Two points are worth mentioning. First, we impose strict concavity of \( u \) in Lemma

\(^{13}\)This assertion holds almost surely with respect to \((\Omega, \mathcal{B}, P)\).
1 and Proposition 1 so that the optimal strategy $C$ given in equation (7) is continuous and the asymptotic distribution $\psi$ is stationary (cf. Lucas and Stokey [1989]). Second, autarky is obviously also an equilibrium of this mechanism. We do not know whether or not there are multiple non-autarkic equilibrium. But given the way that the equilibrium is constructed, it Pareto dominates all other equilibria ex ante.

5. Equilibrium of a laissez-faire monetary mechanism is nearly efficient if agents are sufficiently patient

In this section we show that stationary Markov competitive equilibrium of a laissez-faire monetary mechanism is nearly ex ante Pareto efficient in an environment of sufficiently patient traders. To do so, consider a family of environments that are identical in all respects except for the value $\beta$ of the agents’ discount factor. We will show that, as $\beta$ approaches 1, the equilibria constructed in the proof of Proposition 1—in which each trader’s optimization problem is isomorphic to that of an autarkic agent whose endowment is perfectly storable—are nearly efficient.

The concept of near efficiency that we study is a variant of Debreu’s [1951] coefficient of resource utilization. This measures efficiency in consumption units. A mechanism in an environment is $\delta$-efficient, for $\delta \in (0, 1]$, if it has an equilibrium allocation that all agents would weakly prefer ex ante to the full-risk-sharing allocation of the environment in which the endowment of the actual environment is shrunken to any scalar replica of proportion smaller than $\delta$.

Formally, fix a stochastic process $\theta$, endowment function $e$, and utility function $u$ satisfying the requirements of Proposition 1, so that stationary Markov competitive equilibrium is assured to exist. For $\beta \in (0, 1)$ and $\delta \in (0, 1]$, define $E_{\beta\delta}$ to be the environment with stochastic process $\theta$ in which all agents’ preferences are characterized by utility function $u$ and discount factor $\beta$, and in which each trader $i$ receives endowment $\delta e(\theta_{it})$ at date $t$. Let $r_\delta: \Theta \to \mathbb{R}_+$ be a mapping such that $E[r_\delta(\theta) - \delta e(\theta)] = 0$ and also such that there is a common subgradient of $\{u(r_\delta(\theta), \theta)\}_{\theta \in \Theta}$. The allocation implied by $r_\delta$ is the complete risk sharing allocation in economy $E_{\beta\delta}$, for every $\beta$. For every $\beta$ and $\delta$, define $U_\delta = \sum_{\theta \in \Theta} \pi(\theta) u(r_{\delta}(\theta), \theta)$. $U_\delta/(1 - \beta)$ is the ex ante expected discounted utility of consumption in a full-risk-sharing allocation of environment $E_{\beta\delta}$. Note that the consumption

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14 If $u'(r_{\delta}(\theta), \theta)$ exists for each $t$, then the condition that this derivative has the same value for all $\theta$ is equivalent.
levels \( r_\delta(\theta) \) and the expected utility \( U_\delta \) per period do not depend on \( \beta \). By the assumption of Proposition 1 that each \( u(c, \theta) \) is strictly concave in \( c \), \( \delta < \varepsilon \) implies that \( \forall \theta \ r_\delta(\theta) < r_\varepsilon(\theta) \). Thus, because a strictly concave, increasing function on \( \mathbb{R}_+ \) is strictly increasing, \( \delta < \varepsilon \) implies that \( U_\delta < U_\varepsilon \). Define \( V_\beta \) to be the ex ante expected value of consumption in the stationary Markov competitive equilibrium of the laissez-faire monetary mechanism constructed in the proof of Proposition 1. (That is, \( V_\beta = \mathbb{E}_\psi V(w_0) \), where \( V \) is the value function for the auxiliary problem of Lemma 1 with discount factor \( \beta \).) Then the laissez-faire monetary mechanism in environment \( \mathcal{E}_{\beta_1} \) is \( \delta \)-efficient if \( \delta = \sup \{ \varepsilon | V_\beta \geq U_\varepsilon / (1 - \beta) \} \).

**Proposition 2.** For any \( \delta < 1 \), there is a \( \beta < 1 \) such that the laissez-faire monetary mechanism is an \( \delta \)-efficient mechanism of the environments with discount factors in \( [\beta, 1) \).

**Proof.** We set \( \varepsilon = (1 + \delta)/2 \), and we construct a strategy that asymptotically provides the full-risk-sharing allocation in \( \mathcal{E}_{\beta_\varepsilon} \). The expected discounted utility that this strategy yields is a lower bound for \( V_\beta \), which is the expected discounted utility that an agent’s optimal strategy yields. We prove the proposition by using the strategy to show that, for sufficiently large \( \beta \), the lower bound is sufficiently close to \( U_\varepsilon / (1 - \beta) \) that \( V_\beta \geq U_\delta / (1 - \beta) \).

As in the proof of Lemma 2, we define the strategy in terms of the consumption function that it implies. Define \( \Gamma: \mathbb{R}_+ \times \Theta \to \mathbb{R}_+ \) by \( \Gamma(w, \theta) = \min(w + e(\theta), r_\varepsilon(\theta)) \). That is, the agent attempts to replicate the consumption that he would enjoy in the full-risk-sharing allocation in \( \mathcal{E}_{\beta_\varepsilon} \), subject to the constraint that the laissez-faire monetary mechanism in the actual economy \( \mathcal{E}_{\beta_1} \) places on his choice. The strategy for agent \( i \) implied by this consumption function is that \( M^*_i(w_{i0}, \theta_{i0}, \ldots, w_{it}, \theta_{it}) = \max(0, \Gamma(w_{it}, \theta_{it}) - e(\theta_{it})) \) and \( Z^*_i(w_{i0}, \theta_{i0}, \ldots, w_{it}, \theta_{it}) = \max(0, e(\theta_{it}) - \Gamma(w_{it}, \theta_{it})) \). The wealth-updating rule of the laissez-faire monetary mechanism entails that \( w_{it+1} - w_{it} = (1 - \varepsilon) e(\theta_{it}) + \varepsilon e(\theta_{it}) - \Gamma(w_{it}, \theta_{it}) \).

Define \( v_{it} = (1 - \varepsilon)(e(\theta_{it}) - \mathbb{E}[e(\theta_{it})]) + \varepsilon e(\theta_{it}) - r_\varepsilon(\theta_{it}) \). Note that \( \langle v_{it} \rangle_{t \in \mathbb{N}} \) is i.i.d., \( \mathbb{E}[v_{it}] = 0 \), and \( w_{it+1} - w_{it} \geq v_{it} + (1 - \varepsilon) \mathbb{E}[e(\theta_{it})] \). Applying a law of the iterated logarithm (Breiman [1968], Theorem 13.25) to the sums \( \sum_{\tau \leq t} (-v_{it}) \) establishes that \( \lim_{t \to \infty} w_{it} = \infty \) almost surely. Therefore, almost surely \( \exists \tau \forall t \geq \tau \Gamma(w_{it}, \theta_{it}) = r_\varepsilon(\theta_{it}) \).

There is a number \( \varphi > 0 \) such that \( U_\varepsilon - \varphi b > U_\delta \). By the preceding argument, there is date \( \tau \) such that \( P(\{ \omega | \forall t \geq \tau \Gamma(w_{it}, \theta_{it}) = r_\varepsilon(\theta_{it}) \}) > 1 - \varphi / 2 \).

Let \( D = \{ \omega | \forall t \geq \tau \Gamma(w_{it}, \theta_{it}) = r_\varepsilon(\theta_{it}) \} \). Then, for \( t \geq \tau \),

\[
\mathbb{E}[u(\Gamma(w_{it}, \theta_{it}))] = \int_D u(r_\varepsilon(\theta_{it})) dP + \int_{\Omega \setminus D} u(\Gamma(w_{it}, \theta_{it})) dP
\]
\[
\geq \int_{D} u(r_{\epsilon}(\theta_{\text{it}})) \, dP + \int_{\Omega \setminus D} [u(r_{\epsilon}(\theta_{\text{it}})) - b] \, dP \\
= \mathbb{E}[u(r_{\epsilon}(\theta_{\text{it}}))] - bP(\Omega \setminus D) \\
> U_{\epsilon} - (\varphi/2)b.
\]

Therefore \(V_{\beta} \geq \mathbb{E}[\sum_{t \geq \tau} \beta^t u(\Gamma(w_{\text{it}}, \theta_{\text{it}}))] > \beta^\tau(U_{\epsilon} - (\varphi/2)b)/(1 - \beta)\), so \(V_{\beta} > U_{\delta}/(1 - \beta)\) if \(\beta \geq [(U_{\epsilon} - \varphi b)/(U_{\epsilon} - (\varphi/2)b)]^{1/\tau}\). \(\blacksquare\)

6. When agents are impatient

The approximate efficiency of laissez-faire policy with very patient agents does not preclude other policy or mechanism from being even better. The potential efficiency loss might be large when agents are impatient. In this section, we study two examples within the class of environments discussed above. In the first example environment, there is a nonmonetary mechanism that has a stationary equilibrium allocation that agents prefer ex ante to any stationary equilibrium allocation of any monetary mechanism. In the second example environment, the equilibrium of an inflationary monetary mechanism is efficient while a laissez-faire or deflationary monetary mechanism is not.

6.1. An example where a nonmonetary mechanism Pareto dominates a laissez-faire monetary mechanism

Consider an environment where each agent’s marginal utility fluctuates between high (state \(h\)) and low (state \(l\)) over time according to a Bernoulli(1/2) process, \(\Theta = \{h, l\} \subseteq \mathbb{R}_{+}\) and \(0 < l < h\). An agent receives 8 unit of endowment when the state is \(l\) and nothing when the state is \(h\), that is, \(e(l) = 8\), \(e(h) = 0\). Agents’ satiation level of consumption is 16 units each period. An agent with an individual state \(\theta\) derives period utility

\[
(8) \quad u(c, \theta) = \theta \min(c, 16)
\]

from consuming \(c\) units of endowment.\(^{15}\)

\(^{15}\)In this specification, the utility function is not strictly concave and the agent is satiated at consumption level 1. These simplifying assumptions are not crucial to the main result derived here—that there are environments in which a nonmonetary mechanism has a stationary equilibrium allocation that agents prefer ex ante to any stationary equilibrium of any monetary mechanism—but they considerably simplify its proof. We could define \(u(c, \theta) = \theta \min(c, 16) + f(c)\), where \(f: \mathbb{R}_{+} \rightarrow \mathbb{R}_{+}\) is a strictly concave, increasing function having very small right derivative at 0, and our arguments would remain sound with slight modifications. The utility function so defined would be strictly concave and increasing in consumption in every state.
In this environment, the assumption \( e(h) = 0 \) rules out stationary equilibrium in any stationary deflationary mechanism. The reason is that, if \( w \) had a stationary distribution with support in \( \mathbb{R}_+ \), then all agents in state \( h \) whose values of \( w \) were sufficiently close to the minimum of that support would be driven below that minimum by paying the lump-sum tax \( \tau Q \), which would be a contradiction. To further reduce the set of interesting mechanisms, we make the following assumption,

\[
(l) = \sum_{t=1}^{\infty} \left( \frac{\beta}{2} \right)^t h = \frac{\beta}{2 - \beta} h.
\]

The assumption states that for an agent in state \( l \) at date 0, his marginal utility of consumption \( l \) equals the expected discounted marginal utility of consumption on the first date that \( h \) occurs after date 0.

In stationary equilibrium of a laissez-faire monetary mechanism, this assumption implies that an agent \( i \) in state \( l \) at date \( t \) is indifferent among all \((m_{it}, z_{it})\) that result in \( c_{it} \in [0, 16] \) and \( w_{i(t+1)} \in [0, 16] \). An agent prefers not to accumulate more than 1 unit of real balances (that is, prefers that \( w_{i(t+1)} \leq p_0 = p_{t+1} \)) because the surplus above 1 unit would be spent at the second or subsequent date when the agent is in state \( h \), and the utility from consumption then would be discounted even more steeply than the utility of consumption at the first such date. With a stationary inflationary mechanism, since positive inflation reduces the value of receipt from selling a unit of endowment to something less than one unit of real balances, assumption (9) implies that agents in state \( l \) will consume their endowments rather than selling. That is, autarky is the only stationary equilibrium of a stationary inflationary mechanism.

Now we can focus on laissez-faire monetary mechanisms, the only mechanisms that may have non-autarkic stationary equilibria.\(^{16}\) Without loss of generality, consider a laissez-faire

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\(^{16}\)A question might be raised, why attention should be restricted to stationary equilibria. A possible response (stated by Bewley) is that the equilibrium concept assumes rational expectations (that is, no distinction is drawn between agents’ subjective beliefs about random future quantities and the actual stochastic processes followed by those quantities), and that, as a matter of cognitive psychology, this assumption would be unreasonable in a nonstationary equilibrium. A different response would be to acknowledge that a welfare comparison between the best allocations implemented by arbitrary equilibria of the respective mechanisms would be more satisfactory. We confidently believe that there are no non-autarkic equilibria whatsoever of a stationary inflationary mechanism under assumption (9), but there may well be nonstationary equilibria of a stationary deflationary mechanism in which the economy becomes progressively demonetized over time as agents become unable to pay their lump-sum taxes. As Woodford [1990] has observed, this is a starkly different situation from what Friedman [1969] supposed would obtain. Nevertheless, it would be of interest to study whether or not such a nonstationary equilibrium can welfare dominate the equilibrium of a nonmonetary mechanism that we characterize below.
monetary mechanism where the trading price is normalized to 1. That is, for any \( t \geq 0 \), any stationary-equilibrium profiles of agents’ summary statistics \( w_t \in F \), messages \( m_t \in G \), and endowment contributions \( z_t \in P \), for any agent \( i \),

\[
Y_{it}(w_t, m_t, z_t) = \max(0, \min(m_{it}, w_{it}))
\]

\[
W_{it}(w_t, m_t, z_t) = w_{it} + z_{it} - m_{it}
\]

It can be verified routinely that, under assumption (9), the efficient stationary equilibrium is for agents to accumulate just up to 16 units of money. That is, an agent in state \( l \) sells his endowments twice (if he is in state \( l \) at two consecutive dates) to obtain 16 units of money, and thereafter consumes his endowment, and an agent in state \( h \) purchases as much as he can afford up to satiation level (16 units of consumption). That is,

\[
Z_{it}(w_{i0}, \theta_{i0}, \ldots, w_{it}, \theta_{it}) = \begin{cases} 
\min(16 - w_{it}, 8) & \text{if } \theta_{it} = l \text{ and } w_{it} \in [0, 16) \\
0 & \text{otherwise}
\end{cases}
\]

\[
M_{it}(w_{i0}, \theta_{i0}, \ldots, w_{it}, \theta_{it}) = \begin{cases} 
\min(w_{it}, 16) & \text{if } \theta_{it} = h \\
0 & \text{otherwise}
\end{cases}
\]

At this stationary equilibrium, the support of the money balances distribution is \( \{0, 8, 16\} \).

If an agent’s money balance is off the support, it will get back on it in finite time. For in stationary equilibrium, at the beginning of date \( t > 1 \), \( 1/2 \) of the population was in state \( h \) at date \( t - 1 \) and therefore has \( w_t = 0 \). Another \( 1/4 \) of the population was in state \( l \) at date \( t - 1 \) and state \( h \) at date \( t - 2 \), and therefore has \( w_t = 8 \). The remaining \( 1/4 \) of the population was in state \( l \) at both dates \( t - 1 \) and \( t - 2 \), and so has \( w_t = 16 \). That is, the stationary wealth distribution in equilibrium is

\[
\mu(w_t = 0) = \frac{1}{2} \quad \text{and} \quad \mu(w_t = 8) = \mu(w_t = 16) = \frac{1}{4}.
\]

The value \( \bar{V}(w) \) of having wealth \( w \) at the beginning of date \( t \), and of experiencing \( \theta_t = l \) and \( \theta_t = h \) with equal probability, is specified by the following system of equations.

\[
\bar{V}(0) = \frac{1}{2} \beta \bar{V}(8) + \frac{1}{2} \beta \bar{V}(0)
\]

\[
\bar{V}(8) = \frac{1}{2} \beta \bar{V}(16) + \frac{1}{2} (8h + \beta \bar{V}(0))
\]

\[
\bar{V}(16) = \frac{1}{2} (8l + \beta \bar{V}(16)) + \frac{1}{2} (16h + \beta \bar{V}(0))
\]

Define \( V^* = E_{\mu} \bar{V}(w) \). Then the sum of equations (15) – (17), weighted by the stationary probabilities of the states corresponding to the values on their left sides, can be rewritten
as

(18) \[ V^* = \frac{3h + l}{1 - \beta} \]

Now consider a nonmonetary mechanism. The initial assignment \( w_0 \in F \) is independent of the population process \( \{\theta_{it}\} \) for every \( i \) (as specified in footnote 9), and satisfies \( \mu(\theta_{it} = 0) = \mu(\theta_{it} = 1) = 1/2 \). \( Y \) and \( W \) are defined as follows. At each date, the agent must make a contribution of either 0 or 4 units, or else the mechanism will subsequently treat the agent autarkically. At each date \( t \), the mechanism distributes equally the contributions received to the agents who contribute 0 units and for whom \( w_{it} = 1 \). All agents \( i \) who contribute 0 units at date \( t \) will be assigned \( w_{i(t+1)} = 0 \). All agents \( i \) who contribute 4 units at date \( t \) will be assigned \( w_{i(t+1)} = 1 \). Note that the functionals \( Y_t \) and \( W_t \) depend only on the profiles \( w_t \) and \( z_t \), and not on the message profile \( m_t \), and that the mechanism is stationary.

In the environment of this example, satisfying equation (9), it is routine to verify that a stationary equilibrium of this mechanism is for agents in state \( l \) to contribute 4 units and for agents in state \( h \) to contribute 0 units. Let \( \bar{U}(w) \) be the value in this equilibrium of having wealth \( w \) at the beginning of date \( t \), and of experiencing \( \theta_t = l \) and \( \theta_t = h \) with equal probability. On the support \( \{0, 1\} \) of the stationary distribution of \( w \), this value function is determined by the pair of recursive equations

(19) \[ \bar{U}(0) = \frac{1}{2}(4l + \beta\bar{U}(1)) + \frac{1}{2} \beta\bar{U}(0) \]
(20) \[ \bar{U}(1) = \frac{1}{2}(4l + \beta\bar{U}(1)) + \frac{1}{2}(16h + \beta\bar{U}(0)) \]

Define \( U^* = \bar{U}(0)/2 + \bar{U}(1)/2 \), which is the ex ante value of participation in this stationary equilibrium. Adding equations (19) and (20) and dividing by 2 yields

(21) \[ U^* = \beta U^* + 4h + 2l \]

so that

(22) \[ U^* = \frac{4h + 2l}{1 - \beta} \]

Thus \( U^* > V^* \), so the stationary equilibrium allocation of this nonmonetary mechanism is preferred ex ante to the best stationary equilibrium of the laissez-faire monetary mechanism, which is the only stationary monetary mechanism that possesses a stationary equilibrium.

6.2. An example where inflationary policy Pareto dominates laissez-faire
Consider an environment where agents’ marginal utility fluctuates between high (state \( h \)) and low (state \( l \)) over time, \( \Theta = \{ h, l \} \subseteq \mathbb{R}_+ \) and \( 0 < l < h \), but they all receive a constant endowment \( e(\theta) \equiv e \) for all \( \theta \in \Theta \) every period. For agent \( i \), \( \theta_{it} \) is i.i.d. with a Bernoulli(1/2), that is, the probability of \( \theta_{it} = h \) is 1/2 for all \( t \geq 0 \). Agents have a satiation level of consumption \( \zeta \) each period, \( \zeta > 2e \). An agent with an individual state \( \theta \) derives period utility

\[
 u(c, \theta) = \theta \min(c, \zeta)
\]

from consuming \( c \) units of endowment.

The first-best outcome in this environment is to have agents with low marginal utility transfer all endowment to agents with high marginal utility. Moreover, because utility is linear on \([0, \zeta]\), any such transfer that does not exceed state-\( h \) traders’ satiation levels is efficient. We show that under some parameter restriction, such an outcome can be achieved as an equilibrium of a stationary inflationary monetary mechanism. The efficiency of inflationary policy in this example is fragile. It depends crucially on the local risk-neutrality just mentioned. Nevertheless, it is a robust feature (cf. footnote 14) that this policy is superior to laissez-faire.

Consider a stationary monetary mechanism specified by policy \( \tau \) and trading price normalized to 1. That is, for any \( t \geq 0 \), any profiles of agents’ summary statistics \( w_t \in F \), messages \( m_t \in G \), and endowment contributions \( z_t \in P \), for any agent \( i \),

\[
 Y_{it}(w_t, z_t, m_t) = \max(0, \min(m_{it}, w_{it}))
\]

\[
 W_{it}(w_t, z_t, m_t) = \tau Q + (1 - \tau)(w_{it} + z_{it} - m_{it})
\]

where \( Q = \int_I w_{jt} d\mu \). We are going to show that the following strategy is an equilibrium strategy of the mechanism,

\[
 Z_{it}(w_{i0}, \theta_{i0}, \ldots, w_{it}, \theta_{it}) = \begin{cases} 
 e & \text{if } \theta_{it} = l; \\
 0 & \text{otherwise}
\end{cases}
\]

\[
 M_{it}(w_{i0}, \theta_{i0}, \ldots, w_{it}, \theta_{it}) = \begin{cases} 
 0 & \text{if } \theta_{it} = l \\
 w_{it} & \text{otherwise}
\end{cases}
\]

That is, an agent spends all his money on consumption when marginal utility is high (\( \theta_{it} = h \)), and sells all his endowment \( e \) when his marginal utility is low (\( \theta_{it} = l \)). Such an outcome is efficient.

Following this strategy, agents’ money balances (summary statistics) are concentrated on a set \( \{\alpha_n\}_{n=0}^\infty \), where \( \alpha_n \) is an agent’s money balance after \( n \) consecutive sales since his
last purchase,

\[(28)\] \[\alpha_0 = \tau Q\]

\[(29)\] \[\forall n \geq 1 \quad \alpha_n = \tau Q + (1 - \tau)(\alpha_{n-1} + e).\]

Recursively applying (29), for \(n \geq 1\),

\[(30)\] \[\alpha_n = \frac{1}{\tau} \left[ \tau Q + e(1 - \tau) - (\tau Q + e)(1 - \tau)^{n+1} \right].\]

Given that the environment is stationary, and that agents’ taste shock follows a Bernoulli process, for all \(n \geq 0\), the measure of agents whose money balances are \(\alpha_n\) is

\[(31)\] \[\mu\{w_{it} = \alpha_n\} = \frac{1}{2^{n+1}}.\]

Then

\[(32)\] \[Q = \sum_{n=0}^{\infty} \alpha_n \mu\{w_{it} = \alpha_n\} = \frac{1}{\tau} \left[ \tau Q + e(1 - \tau) - (\tau Q + e)\frac{1 - \tau}{1 + \tau} \right].\]

Solving \(Q\) from (32), we have

\[(33)\] \[Q = e.\]

That is, at this equilibrium, aggregate real money balance at any date (which is also per capita real money balance given that the measure of agent is 1) equals to an agent’s endowment. By (30) and (32), for \(\tau \in (0, 1)\),

\[(34)\] \[\lim_{n \to \infty} \alpha_n = \frac{e}{\tau}.\]

Given the satiation level \(\zeta\), the optimality of strategy for \(\theta_{it} = h\) (spending all money on consumption) requires that \(e + \alpha_n \leq \zeta\) for all \(n \geq 0\). Therefore, a necessary condition for the optimality of strategy \((M, Z)\) is

\[(35)\] \[e + \frac{e}{\tau} \leq \zeta.\]

The value function on \(\{\alpha_n\}_{n=0}^{\infty}\) is defined as follows. For all \(n \geq 0\),

\[(36)\] \[V(\alpha_n) = \frac{1}{2} \left( h(e + \alpha_n) + \beta V(\alpha_0) \right) + \frac{1}{2} V(\alpha_{n+1}).\]

The solution to this system of equations can be expressed recursively as follows.

\[(37)\] \[V(\alpha_0) = \frac{h e(1 + \tau)}{1 - \beta/2 - \beta(1 - \tau)}\]

\[(38)\] \[\forall n \geq 1 \quad V(\alpha_n) = V(\alpha_{n-1}) + \frac{h e(1 + \tau)(1 - \tau)^n}{2 - \beta(1 - \tau)}.\]
By (30), (33) and (38),

\[
\frac{V(\alpha_n) - V(\alpha_{n-1})}{\alpha_n - \alpha_{n-1}} = \frac{he(1 + \tau)(1 - \tau)^n}{2 - \beta(1 - \tau)} \left/ \left( e(1 + \tau)(1 - \tau)^n \right) \right. = \frac{h}{2 - \beta(1 - \tau)}
\]

which is a constant. Hence, the value function \( V \) is affine on \([e\tau, e/\tau])\) with slope given by (39).

Given the value function, we can verify that the conjectured strategy \((M, Z)\) given in (26) and (27) as equilibrium strategy.

**Proposition 3.** Strategy \((M, Z)\) given in (26) and (27) is optimal if the parameters of the model \(\beta, l, h\) and policy variable \(\tau\) satisfy the following condition,

\[
\frac{e}{\zeta - e} \leq \tau \leq \frac{\beta(h + l) - 2l}{\beta(h + l)}.
\]

**Proof.** The first inequality of condition (40) is a restatement of condition (35). We need only to show the second half of the condition.

When \(\theta_{it} = h\), strategy \((M, Z)\) specifies the optimal net trade to be \(x^* = -w_{it}\). This is optimal if for any \(\varepsilon > 0\), and any \(x \in [-w_{it}, e]\) such that \(x - \varepsilon \in [-w_{it}, e]\), the expected value of net trade \(x\) is lower than that of \(x - \varepsilon\), that is,

\[
h(e - x) + \beta V(\tau e + (1 - \tau)(w_{it} + x)) \leq h(e - x + \varepsilon) + \beta V(\tau e + (1 - \tau)(w_{it} + x - \varepsilon)).
\]

Given that the value function \(V\) is affine with slope \(h/(2 - \beta(1 - \tau))\), this inequality is equivalent to

\[
\frac{\beta h}{2 - \beta(1 - \tau)} \leq \frac{h\varepsilon}{(1 - \tau)\varepsilon}
\]

which always holds. That is, given the first half of condition (40), \(x^* = -w_{it}\) when \(\theta_{it} = h\) is optimal.

When \(\theta_{it} = l\), strategy \((M, Z)\) specifies the optimal net trade to be \(x^* = e\). This is optimal if for any \(\varepsilon > 0\), any \(x \in [-w_{it}, e]\) such that \(x + \varepsilon \in [-w_{it}, e]\), the expected value of net trade \(x\) is lower than that of \(x + \varepsilon\), that is,

\[
l(e - x) + \beta V(\tau e + (1 - \tau)(w_{it} + x)) \leq l(e - x - \varepsilon) + \beta V(\tau e + (1 - \tau)(w_{it} + x + \varepsilon)).
\]

This inequality is equivalent to

\[
\frac{l\varepsilon}{(1 - \tau)\varepsilon} \leq \frac{\beta h}{2 - \beta(1 - \tau)}
\]
or the second half of condition (40). That is, $x^* = e$ when $\theta_d = l$ is optimal if the second half of condition (40) is satisfied.

By Proposition 3, the efficient allocation in this environment is achieved by the equilibrium of a stationary inflationary monetary mechanism since policy $\tau > e/(\zeta - e) > 0$. Any policy with $\tau \leq 0$, i.e., laissez-faire or deflationary monetary mechanism, would not accomplish the task. With an inflationary policy, all agents’ money balances are bounded by $e/\tau$ given that they are constantly inflated away at a rate $\tau$. So “rich” people can never get too rich to not sell. If $\tau \leq 0$, however, selling whenever an agent’s marginal utility is low is no longer an optimal strategy. Let $\hat{t}$ be the smallest $t$ such that $\beta^t h < l$, so for all $t \geq \hat{t}$, the discounted marginal utility of consumption in state $h$ after $t$ periods is lower than the marginal utility of consuming in today’s $l$ state. Then when an agent in state $l$ today has money balances $t(\zeta - e)$, $t \geq \hat{t}$, he will consume rather than selling his endowment, contrary to strategy $(M, Z)$ given in (26) and (27), as well as the prescription to achieve the efficient allocation.

7. Conclusion

We consider a class of environments where there is a stringent restriction on the amount of information that can be kept regarding the history of each agent, where an agent’s endowment cannot be taken from him forcibly or by threat of nonpecuniary punishment, and where an agent’s current characteristics are his private information. We suggest that this class of environments formalizes the assumptions under which, according to previous conjectures, spot trade using fiat money can be an exactly or approximately efficient allocation mechanism if monetary policy is set appropriately. Within this class of environments, we provide an explicit definition of a monetary mechanism and particularly of a monetary mechanism governed by laissez-faire policy. We show that a laissez-faire monetary mechanism is nearly efficient, in terms of a criterion in the spirit of Debreu’s coefficient of resource utilization for ex ante Pareto efficiency, in an environment within our class where agents are sufficiently patient. We also provide examples showing that, in an environment within our class where agents are impatient, (1) a stationary equilibrium of a nonmonetary mechanism can be preferred ex ante to any stationary equilibrium of any stationary monetary mechanism, and (2) an inflationary monetary mechanism can be preferred ex ante to any laissez-faire or deflationary monetary mechanism.
References


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